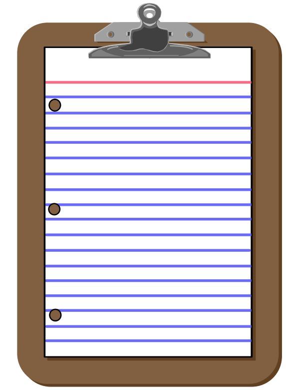
Feed Sideward

Understanding Biological Rhythms

Jim Holte

1/15/2002

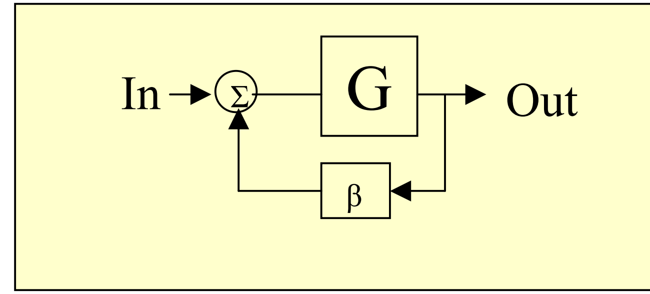
Sessions

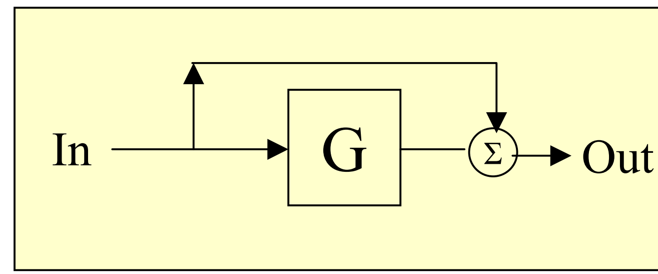


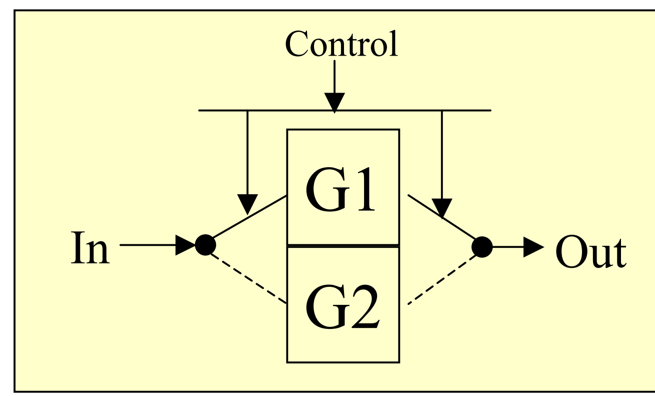
- Session 1 - Feed Sideward – Concepts and Examples, *1/15*
- Session 2 – Feed Sideward – Applications to Biological & Biomedical Systems, *1/31*
- Session 3 – Chronobiology, *2/12*
  Franz Hallberg and Germaine Cornalissen

Feed Sideward

Terms	**Simple Example**
• Feed Back	Reinvesting dividends
• Feed Foreward	Setting money aside
• Feed Sideward	Moving money to another account







Introduction

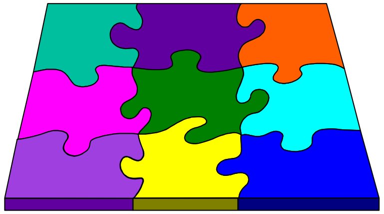
Feed Sideward is a coupling that shifts resources from one subsystem to another

- Feed Sideward #1 – feeds ***values of other variables*** into the specified variable
- Feed Sideward #2 – feeds ***changes of parameters*** into the specified variable. (time varying parameters)
- Feed Sideward #3 – feeds ***changes of topology*** by switch operations (switched systems)

Tool for global analysis

especially useful for biological systems

Phase Space



- Laws of the physical world
- Ordinary differential equations
- Visualization of Solutions
- Understanding

Phase Space

The Lotka-Volterra Equations for
Predator-Prey Systems

H' = b*H - a*H*P
P' = -d*P + c*H*P

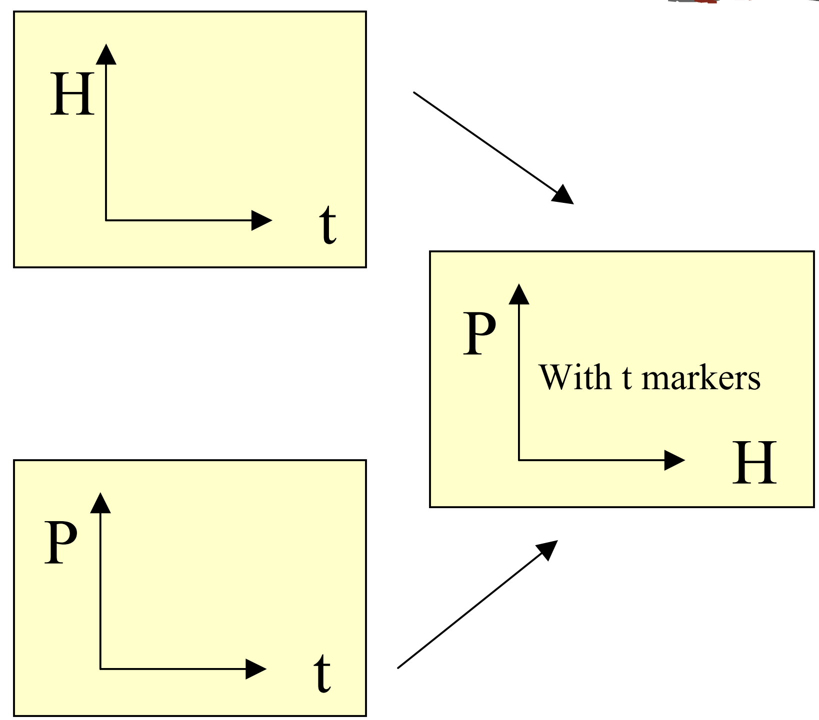
H = prey abundance, P = predator

<u>Set the parameters</u>
b = 2 growth coefficient of prey

d = 1 growth coefficient of predators

a = 1 rate of capture of prey per predator per unit time

c = 1 rate of "conversion" of prey to predators per unit time per predator.



Source: ODE Architect, Wiley, 1999

Phase Space

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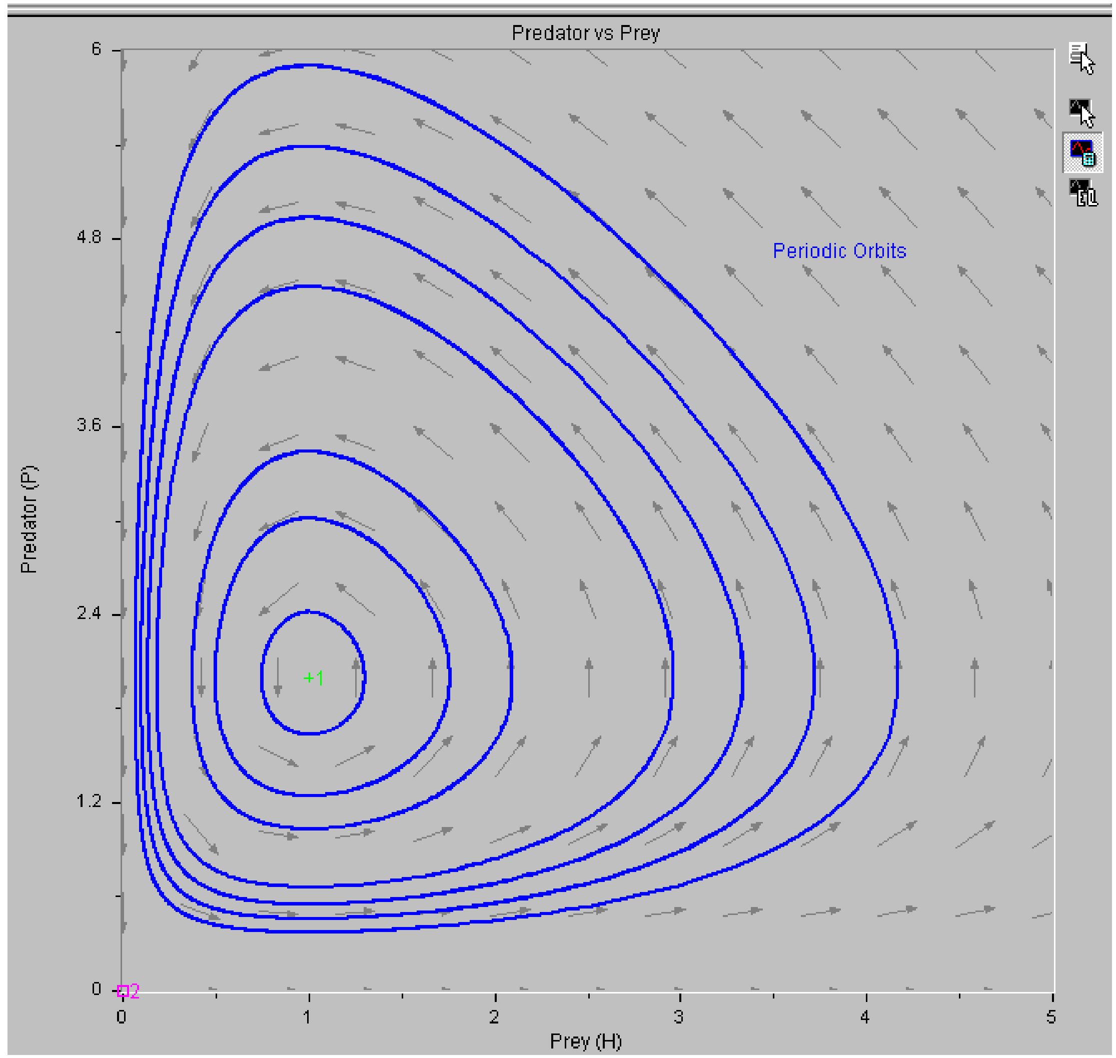
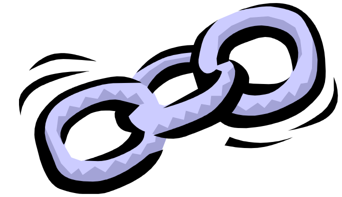


Source: ODE Architect, Wiley, 1999

Coupled Oscillators Model



- x and y represent the "phases" of two oscillators.

  Think of x and y:
  - angular positions of two "particles"
  - moving around the unit circle
- a1 = 0
  - x has constant angular rate
- a2 = 0
  - y has constant angular rate.
- Coupling when a1 or a2 non-zero

Source: ODE Architect, Wiley, 1999

Example
Uncoupled Oscillators

The Tortoise and the Hare

x' = w1 + a1*sin(y - x)

y' = w2 + a2*sin(x - y)

u = (x mod(2*pi)) //Wrap around the

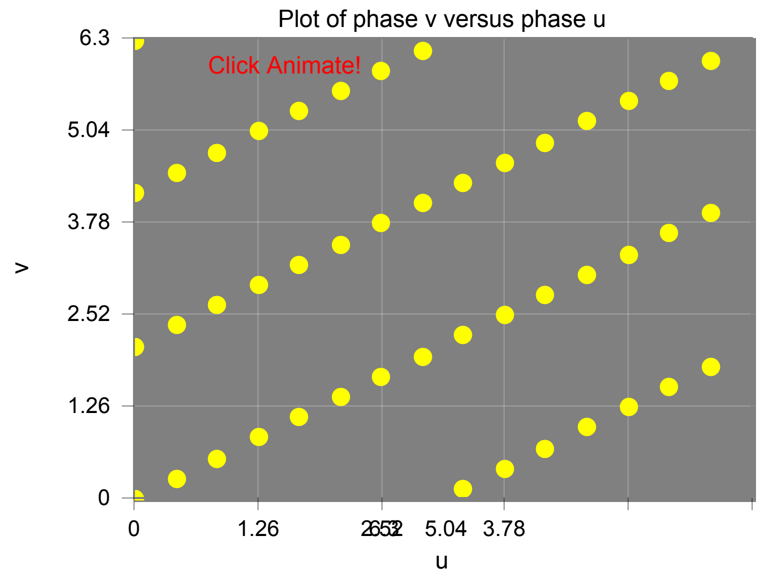
v = (y mod(2*pi)) //unit circle

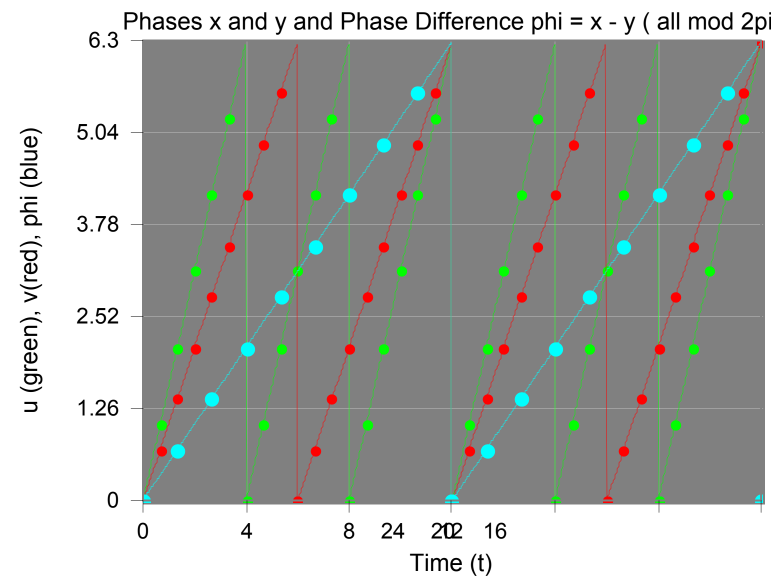
phi = (x - y)mod(2*pi)

<u>Set the parameters</u>

a1 = 0.0; a2 = 0.0

w1 = pi/2; w2 = pi/3





Source: ODE Architect, Wiley, 1999

Example
Coupled Oscillators

Coupled Oscillators:

The Tortoise and the Hare

x' = w1 + a1*sin(y - x)

y' = w2 + a2*sin(x - y)

u = (x mod(2*pi)) //Wrap around the

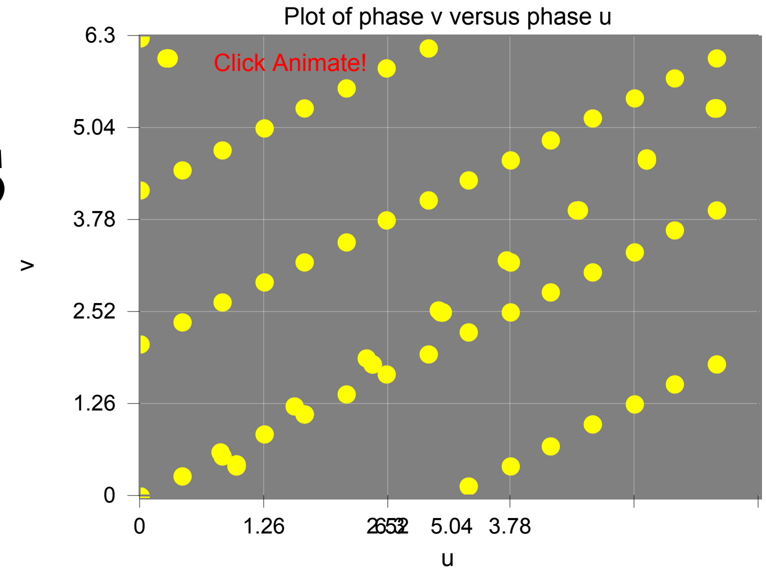
v = (y mod(2*pi)) //unit circle

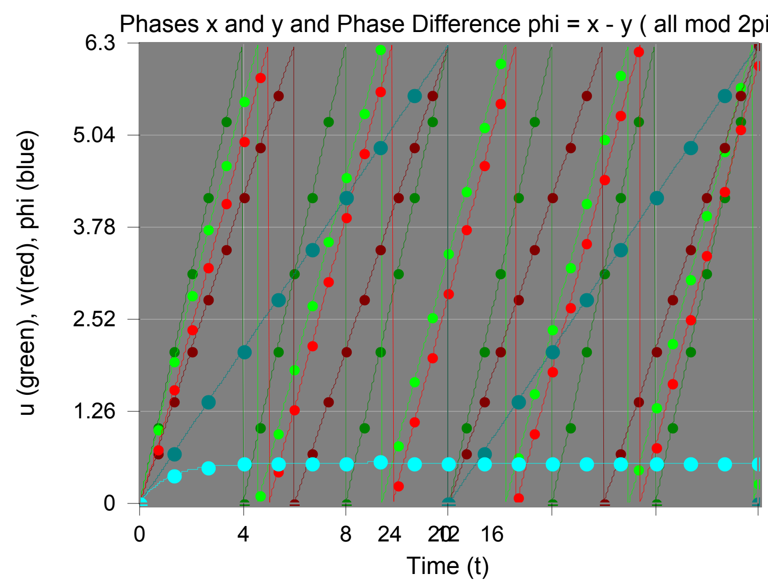
phi = (x - y)mod(2*pi)

<u>Set the parameters</u>

a1 = 0.5; a2 = 0.5

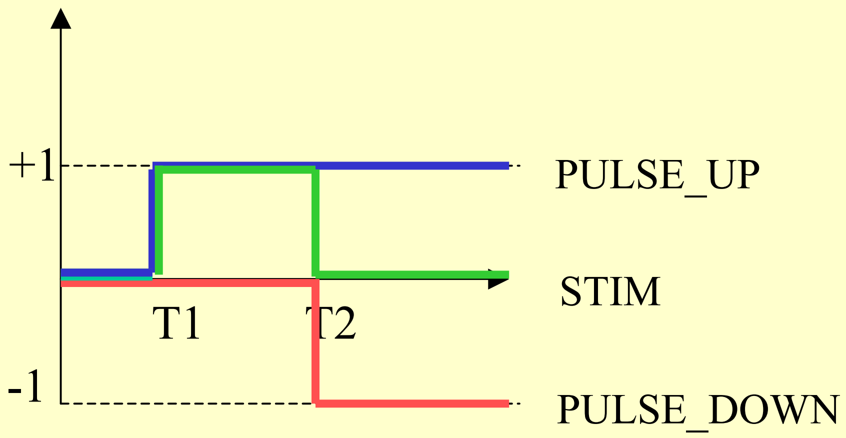
w1 = pi/2; w2 = pi/3





Source: ODE Architect, Wiley, 1999

Phase Resetting




```
FUNCTION PULSE_UP(t, T1, STIM_H)
IF (t >= T1) THEN
  PULSE_UP = STIM_H
ELSE
  PULSE_UP = 0
ENDIF
RETURN PULSE_UP
END
```

```
FUNCTION STIM(t,T1,T2,STIM_L,STIM_H)
STIM = PULSE_UP(t, T1, STIM_H) +
PULSE_DOWN(t, T2, STIM_L)
RETURN STIM
END
```

```
FUNCTION PULSE_DOWN(t,T2,STIM_L)
IF (t <= T2) THEN
  PULSE_DOWN = 0
ELSE
  PULSE_DOWN = STIM_L
ENDIF
RETURN PULSE_DOWN
END
```

Example
Phase Resetting

Theta' = 1 + STIM(t,T1,T2,STIM_L,STIM_H)*cos(2*Theta)

T1 = 4

T2 = 4

STIM_L = -1

STIM_H = +1

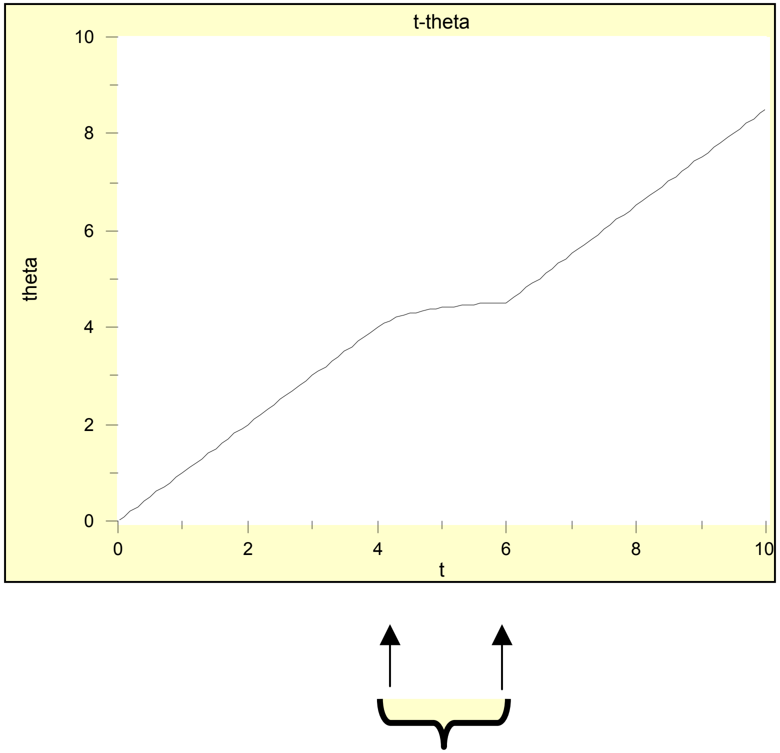
Theta' = 1 + STIM(t,T1,T2,STIM_L,STIM_H)*cos(2*Theta)

T1 = 4

T2 = 6

STIM_L = -1

STIM_H = +1



Source: ODE Architect, Wiley, 1999

Oscillator Entrainment

- x and y represent the "phases“ of two oscillators.

  Think of x and y:
  - angular positions of two "particles"
  - moving around the unit circle
- a1 = 0
  - ➢ x has constant angular rate
- a2 = 0
  - ➢ y has constant angular rate.
- Coupling when a1 & a2 non-zero

- Entrainment occurs when the **coupling causes**
  - angular rate of **x** to
  - approach angular rate of **y**
- x and y generally differ
  - Typical for Chronobiology
- Dominant oscillator **‘entrains’** the other

Source: ODE Architect, Wiley, 1999

Oscillator Entrainment Example

x' = w1 + a1*sin(y - x)

y' = w2 + a2*sin(x - y)

u = (x mod(2*pi)) //Wrap around the

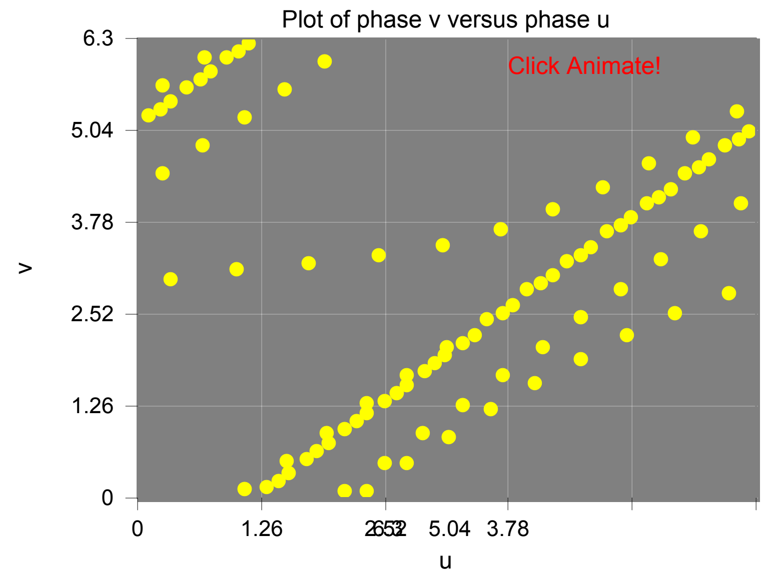
v = (y mod(2*pi)) //unit circle

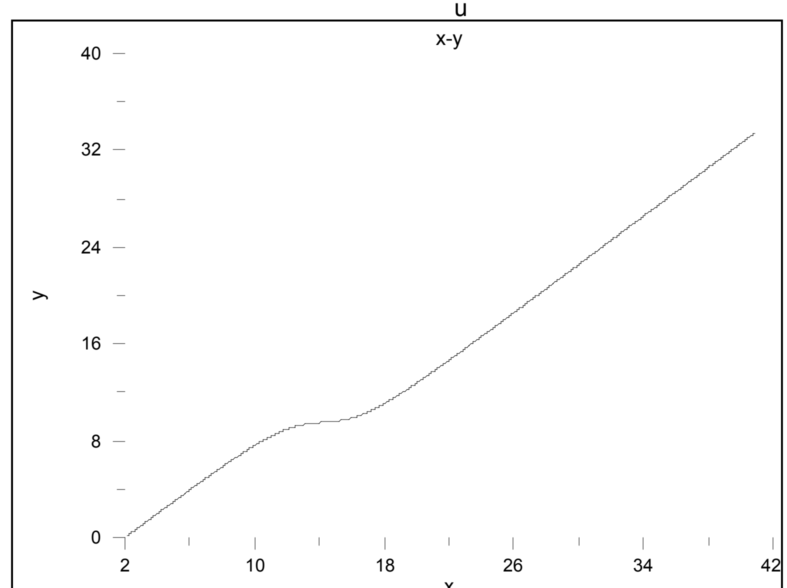
phi = (x - y)mod(2*pi)

Set the parameters

a1 =.0775*pi; a2 =.075*pi

w1 = pi/4; w2 = pi/4 - .14*pi





Source: ODE Architect, Wiley, 1999

Singularities

```
r' = -(r-0)*(r-1/2)*(r-1) - a*STIM(t,T1,T2,STIM_L,STIM_H)

theta' = 1

x = r*cos(theta)

y = r*sin(theta)


T1 = 4

T2 = 6

a=0.0

STIM_L = -1

STIM_H = +1
```

Example - Singularities

Run	r	a	Commment
---	---	---	---------
#1	1.25	0	approaches r=1
#2	1.0	0	stable periodic orbit
#3	0.75	0	approaches r=1
#4	0.5	0	unstable periodic orbit
#5	0.25	0	approaches r=0
#6	0	0	stable periodic orbit
#7	0.75	0.4	starts in r=1 domain,
			STIM moves it to r=0 domain

```
r' = -(r-0)*(r-1/2)*(r-1) - a*STIM(t,T1,T2,STIM_L,STIM_H)

theta' = 1

x = r*cos(theta)

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T1 = 4

T2 = 6

a=0.0

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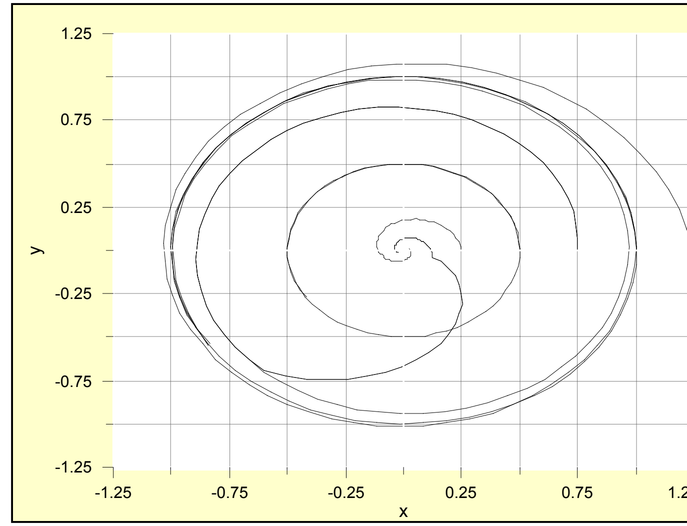
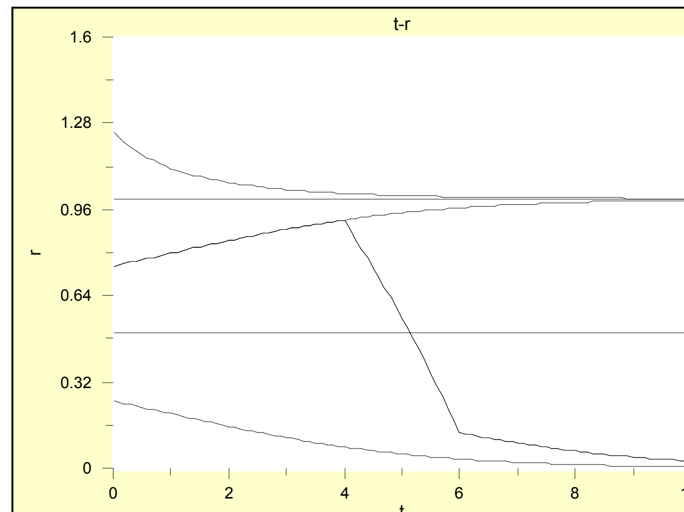
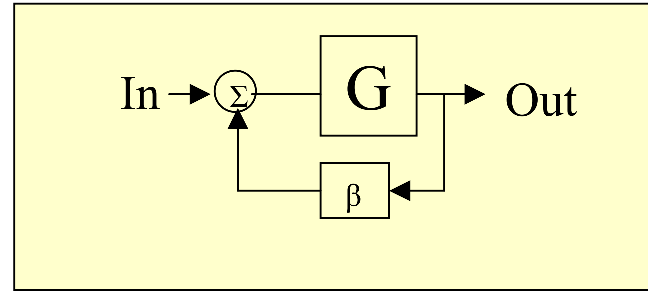
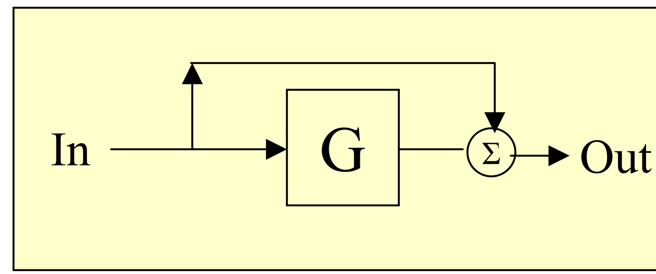
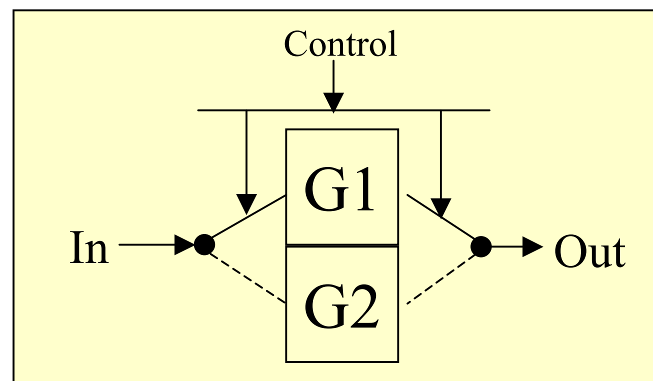




Source: Holte & Nolley, 2002

Feed Sideward

Terms	**Simple Example**	
• Feed Back	Reinvesting dividends	
• Feed Foreward	Setting money aside	
• Feed Sideward	Moving money to another account	

Jim Holte
University of Minnesota

Feed Sideward Example

The Oregonator Model for Chemical Oscillations

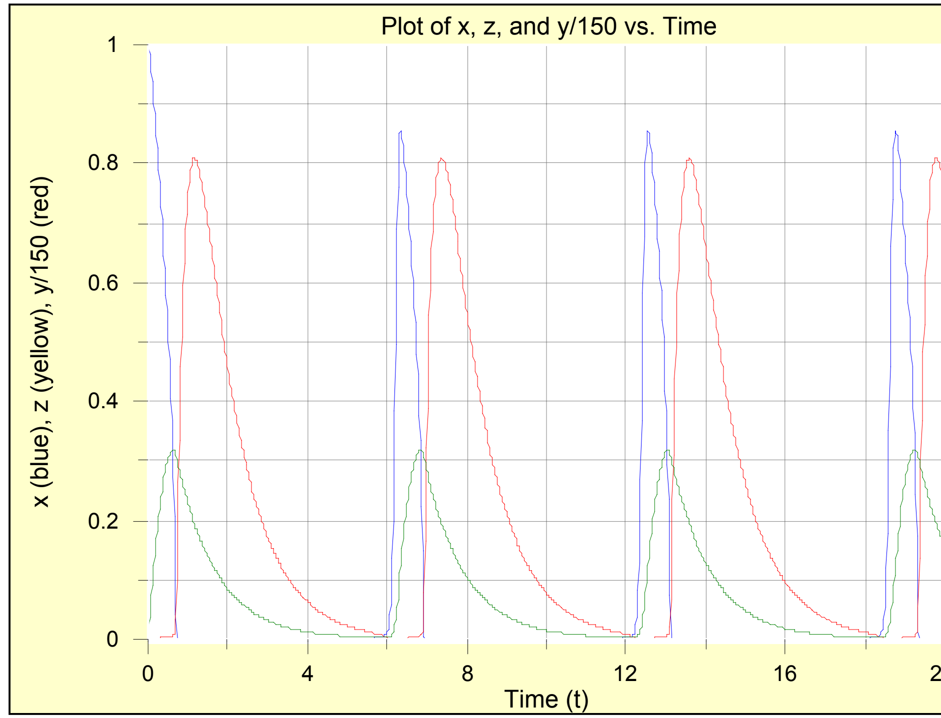
$x' = a1*(a3*\mathbf{y} - x*\mathbf{y} + x*(1-x))$

$y' = a2*(-a3*y - \mathbf{x}*y + f*\mathbf{z})$

$z' = x - z$

$smally = y/150$

$a1 = 25; a3 = 0.0008; a2 = 2500; f = 1$



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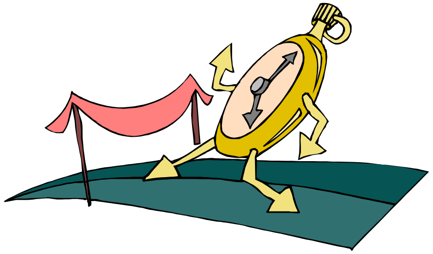
Summary



Feed Sideward is a coupling that shifts resources from one subsystem to another

- Feed Sideward #1 – feeds ***values of other variables*** into the specified variable
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- Feed Sideward #3 – feeds ***changes of topology*** by switch operations (switched systems)

Tool for global analysis

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Next Session

- Session 1 - Feed Sideward – Concepts and Examples, *1/15*
- Session 2 – Feed Sideward – Applications to Biological & Biomedical Systems, *1/31*
- Session 3 – Chronobiology, *2/12*
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Backup

Feed Sideward - Topics (60 min)

Session 1 (14 slides)

- Background Concepts & Examples
  - Phase Space (1 slide)
  - Singularities (2 slides) *
  - Coupled Oscillators (2 slides)
  - Phase Resetting (2 slides) *
  - Oscillator Entrainment (1 slide)
- Feed Sideward as modulation (3 slides) **
- Summary (1 slide)

Session 2 (12 slides)

- Applications to Biological Systems
  - Circadian & other Rhythms (2 slides)
    - Model & Simulation Result (2 slides)
- Applications to Biomedical Systems
  - Blood Pressure Application (2 slides)
    - Model & Simulation Result (2 slides)
- Summary (1 slide)
- Segue to Chronobiology (1 slide)